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ANALYTICAL SOLUTION TO MHD FLOW AND HEAT TRANSFER OF VISCO-ELASTIC FLUID OVER AN EXPONENTIALLY STRETCHING SHEET IN POROUS MEDIA WITH VISCOUS DISSIPATION

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ABSTRACT

In this paper, we study the heat transfer of MHD visco-elastic fluid flow through porous medium over an exponentially stretching sheet with viscous dissipation. By using suitable similarity transformations the governing partial differential equations are converted into non linear ordinary differential equations. These equations are analytically solved by using Homotopy Analysis Method (HAM). The convergence of the series solution which is obtained in the form of infinite series is discussed explicitly. The effect of various physical parameters like Magnetic parameter, Prandtl number, Eckert number, porosity parameter and visco-elastic parameter on velocity and temperature are discussed and displayed graphically. By increasing Prandtl number, the temperature decreases whereas it increases by increasing Eckert number.

KEYWORDS: Visco-Elastic Fluid, MHD, Porous Medium, Viscous Dissipation, Exponential Stretching Sheet & HAM

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Original Article

1. INTRODUCTION

The study of the boundary layer flow and heat transfer in viscoelastic fluids has ever increasing applications in engineering and manufacturing processes. Sakiadis [1] was the first to investigate boundary layer flow on continuous surfaces of finite length. Crane [2] extended this problem to a stretching sheet whose velocity is proportional to the distance from the slit which occurs in drawing of plastic sheet. P. S. Gupta and A. S. Gupta [3] analyzed momentum, heat and mass transfer of viscous fluid over a stretching sheet subject to suction/blowing. With this motivation Rajgopal et al. [4] examined the visco-elastic fluid flow over a stretching sheet.

H. I. Anderson [5] studied the visco-elastic fluid flow over a stretching sheet in presence of transverse magnetic field. Later, M. I. Char [6] analyzed the heat transfer characteristics of an electrically conducting fluid in presence of transverse magnetic field. By taking into account the mass of the chemically reactive species, Sujith Kumar khan et al. [7] studied the heat and mass transfer of MHD visco-elastic fluid flow over a porous stretching sheet with viscous dissipation. P. S. Datti et al. [8] studied the effect of radiation of MHD visco-elastic fluid flow over a non iso- thermal stretching sheet with internal heat generation/absorption. Rafael Cortell [9] analyzed the flow and heat transfer of an electrically conducting visco-elastic fluid with viscous dissipation. M. S. Abel et al. [10] studied the visco-elastic fluid flow and heat transfer with viscous dissipation and non-uniform heat source.

All the above investigations are restricted to flow and heat transfer problems in non porous medium. The heat transfer in porous medium is carried out in many metallurgical processes. In these processes the filaments and continuous strips are cooled by drawing them through quiescent fluid. In the process of drawing, sometimes these strips are stretched. The rate of cooling can be controlled by drawing these strips in porous medium and the final product is obtained with desired accuracy. In this view, Vajravelu [11] studied the flow and heat transfer of viscous fluid in a saturated porous medium over an impermeable stretching sheet. Subhas et al. [12] studied the flow and heat transfer of visco-elastic fluid in porous medium over a stretching sheet. Mahantesh et al. [13] analyzed the heat transfer characteristics of visco-elastic fluid in porous medium over a stretching surface.

Firstly, S. J. Liao [14] introduced Homotopy Analysis Method (HAM). Further it is developed and improved to solve non-linear problems. Subsequently, Liao [15] analyzed two branches of solutions to boundary layer flows over a stretched impermeable wall by using HAM method. Sajid et al. [16] applied Homotopy Analysis Method to discuss the analytic solution for MHD viscous flow due to a shrinking sheet.

However, all these studies are confined to flow and heat transfer characteristics over a stretching sheet. By considering exponential stretching surface, Magyari et al. [17] described the heat and mass transfer characteristics of viscous fluid. Elbashbeshy [18] analyzed the heat transfer over exponential stretching surface with suction. Sajid et. al [19] studied flow and heat transfer of viscous fluid over an exponentially stretching sheet with radiation. Swati Mukhopadhyay [20] studied flow and heat transfer MHD viscous fluid over exponential stretching sheet in thermally stratified medium. Sujith Kumar Khan et al. [21] examined flow and heat transfer characteristics of visco-elastic fluid over an exponential stretching sheet. Hymavathi et al. [22] studied the momentum and heat transfer of MHD visco-elastic fluid over an exponential stretching sheet through porous medium by using Homotopy Analysis Method.

The main objective of this paper is to study the flow and heat transfer characteristics of an electrically conducting visco-elastic fluid over an exponentially stretching sheet through porous medium with viscous dissipation by using Homotopy Analysis Method (HAM). The effect of various physical parameters like visco-elastic parameter (k_1), porosity parameter (k_2), Magnetic parameter (M), Prandtl number (Pr) and Eckert number (Ec) on velocity and temperature are discussed through graphical representation. The results are compared with the available results in the literature and are seen in good agreement.

2. MATHEMATICAL FORMULATION

Consider an electrically conducting, steady, two dimensional boundary layer flow and heat transfer of viscoelastic fluid over an exponentially stretching sheet in porous medium. The flow is confined to y>0, x-axis is taken along the stretching surface in the direction of motion and y-axis is normal to it. By simultaneous application of equal and opposite forces along x-axis keeping the origin fixed the flow is generated as a consequence of exponential stretching of the sheet. A uniform magnetic field of strength B_0 is applied normal to the stretching surface. The flow and heat transfer characteristics under the boundary layer approximations are governed by the equations.

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\upsilon\frac{\partial^2 u}{\partial y^2}-k_0\left\{u\frac{\partial^3 u}{\partial x\partial y^2}+v\frac{\partial^3 u}{\partial y^3}+\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2}-\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x\partial y}\right\}-\frac{\upsilon}{k^*}u-\frac{\sigma B_0{}^2 u}{\rho} \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho C_p}\frac{\partial^2 T}{\partial y^2}+\frac{\mu}{\rho C_p}\left[\left(\frac{u^2}{k^*}\right)+\left(\frac{\partial u}{\partial y}\right)^2\right] \tag{3}$$

Where u and v are the velocities in x and y directions respectively, T is the temperature of the fluid, ρ is the density of the fluid, k^* is the permeability coefficient of the porous medium, σ is electrical conductivity of the fluid, C_p is the specific heat at constant pressure,

The boundary conditions under the consideration are given by

$$\begin{aligned} &u=U_w(x)=U_0e^{x/l},\ v=0,\quad T=T_w && at\ y=0\\ &u=0,\ u_y=0.\ T=T_\infty+T_0e^{x/l} && as\ y\to\infty \end{aligned} \tag{4}$$

Where U_o, T_0, l are the reference velocity, the reference temperature and the reference length respectively. T_∞ is the temperature far away from the stretching sheet.

Equation of continuity (1) is identically satisfied if we choose the stream function ψ such that

$$u=\frac{\partial\psi}{\partial y},\ v=-\frac{\partial\psi}{\partial x} \tag{5}$$

The momentum and energy equations can be transformed into the corresponding ordinary differential equations by introducing the similarity transformations:

$$\eta=y\sqrt{\frac{U_0}{2\upsilon l}}\ e^{x/2l} \tag{6}$$

$$\psi(x,y)=\sqrt{2\upsilon l U_0}\,f(x.\eta)e^{x/2l},\qquad \theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty} \tag{7}$$

Where η is the similarity variable f is the dimensionless stream function, $\theta(\eta)$ be the dimensionless temperature

The momentum and energy equations are transformed to

$$f'''-2f'^2+ff''-k_1\left\{3f'f'''-\frac{1}{2}ff^{iv}-\frac{3}{2}f''^2\right\}-(M+k_2)f'=0 \tag{8}$$

$$\theta''+\Pr\left(f\theta'-2f'\theta\right)+k_2\Pr Ecf'^2+\Pr Ecf''^2=0 \tag{9}$$

With the boundary conditions

$$\begin{array}{llll} f=0, & f'=1, & \theta=1 & at\ \eta=0 \\ f'=0, & f''=0, & \theta=0 & as\ \eta\to\infty \end{array} \tag{10}$$

Where $k_1=\dfrac{\alpha_1 U_w}{\rho \upsilon l}$ is the dimensionless visco-elastic parameter, $k_2=\dfrac{2\upsilon l}{k^* U_w}$ is the porosity parameter, $M=\dfrac{2\sigma B_0^2 l}{\rho U_0 e^{x/l}}$ is the magnetic parameter, $Ec=\dfrac{u_w{}^2}{C_p(T-T_\infty)}$ is the Eckert number, $S=\dfrac{v_0}{u_w\sqrt{2U_0 l}}$ is the suction parameter and $\Pr=\dfrac{\upsilon}{\alpha}$ is the Prandtl number.

3. HOMOTOPY ANALYSIS SOLUTION

In this section, we employ HAM to solve the equation (8) and (9) subject to the boundary conditions (10). Let us choose the initial guesses f_0, θ_0 as

$$f_0(\eta)=1-e^{-\eta} \tag{11}$$

$$\theta_0(\eta)=e^{-\eta} \tag{12}$$

The linear operator is selected as

$$L(f)=f'''-f' \tag{13}$$

$$L(\theta)=\theta''-\theta \tag{14}$$

Which have the following property

$$L_f[C_1+C_2e^{\eta}+C_3e^{-\eta}]=0 \tag{15}$$

$$L_\theta[C_4e^{\eta}+C_5e^{-\eta}]=0 \tag{16}$$

Where C_i $(i=1,2,3,4,5)$ are the arbitrary constants.

If $p\in[0,1]$ is the embedding parameter, $\hbar$ is the non zero auxiliary parameter and $H(\eta)$ is the auxiliary function, then we construct the following zero-th ordinary deformation equation:

$$(1-p)L(\tilde{f}(\eta;p)-f_0(\eta))=p\hbar_1H_1(\eta)N_1[\tilde{f}(\eta;p)] \tag{17}$$

$$(1-p)L(\tilde{\theta}(\eta;p)-\theta_0(\eta))=p\hbar_2H_2(\eta)N_2[\tilde{f}(\eta;p),\tilde{\theta}(\eta;p)] \tag{18}$$

Subject to the boundary conditions

$$\tilde{f}(0;p)=0 \quad \tilde{f}'(0;p)=1 \quad \tilde{f}'(\infty;p)=0$$

$$\tilde{f}''(\infty, p) = 0 \qquad \tilde{\theta}(0; p) = 1 \qquad \tilde{\theta}'(\infty; p) = 0 \tag{19}$$

We define non-linear operator as

$$N_1(\tilde{f}(\eta; p)) = \frac{\partial^3 f}{\partial \eta^3} - 2\left(\frac{\partial f}{\partial \eta}\right)^2 + \frac{\partial^2 f}{\partial \eta^2} f - k_1\left(3\frac{\partial f}{\partial \eta}\frac{\partial^3 f}{\partial \eta^3} - \frac{1}{2} f \frac{\partial^4 f}{\partial \eta^4} - \frac{3}{2}\left(\frac{\partial^2 f}{\partial \eta^2}\right)^2\right) - (k_2 + M)\frac{\partial f}{\partial \eta} \tag{20}$$

$$N_2\left(\tilde{f}(\eta, q), \tilde{\theta}(\eta, q)\right) = \frac{\partial^2 \theta}{\partial \eta^2} + \Pr\left(f\frac{\partial \theta}{\partial \eta}\right) - 2\Pr\frac{\partial f}{\partial \eta}\theta + Ec\Pr k_2\left(\frac{\partial f}{\partial \eta}\right)^2 + Ec\Pr\left(\frac{\partial^2 f}{\partial \eta^2}\right)^2 \tag{21}$$

For $p = 0 \quad and \quad p = 1$, we have

$$f(\eta, 0) = f_0(\eta), \qquad f(\eta, 1) = f(\eta)$$

$$\theta(\eta, p) = \theta_0(\eta) \qquad \theta(\eta, 1) = \theta(\eta) \tag{22}$$

Thus as p increases from 0 to 1, $\tilde{f}(\eta; p)$ varies from $\tilde{f}(\eta; 0)$ *to* $\tilde{f}(\eta)$ and $\tilde{\theta}(\eta; p)$ varies from $\tilde{\theta}(\eta; 0)$ *to* $\tilde{\theta}(\eta)$. Now, expanding $\tilde{f}(\eta, p), \tilde{\theta}(\eta, p)$ in Taylor's series with respect to p, we have

$$\tilde{f}(\eta, p) = f_0(\eta) + \sum_{m=1}^{\infty} f_m(\eta) p^m \tag{23}$$

$$\tilde{\theta}(\eta, p) = \theta_0(\eta) + \sum_{m=1}^{\infty} \theta_m(\eta) p^m \tag{24}$$

Where

$$f_m(\eta) = \frac{1}{m!}\frac{\partial^m \tilde{f}(\eta, p)}{\partial p^m}\bigg|_{p=0} \tag{25}$$

$$\theta_m(\eta) = \frac{1}{m!}\frac{\partial^m \tilde{\theta}(\eta, p)}{\partial p^m}\bigg|_{p=0} \tag{26}$$

If the initial approximations, auxiliary linear operators and non zero auxiliary parameters are chosen in such a way that the series (23) and (24) converges at p=1, then

$$f(\eta) = f_0(\eta) + \sum_{m=1}^{\infty} f_m(\eta) \tag{27}$$

$$\theta(\eta) = \theta_0(\eta) + \sum_{m=1}^{\infty} \theta_m(\eta) \tag{28}$$

Differentiating equations (17) and (18) ‘m’ times with respect to ‘p’, setting p=0 and finally diving with m!, we get the mth-order deformation equations are as follows:

$$L_f[f_m(\eta)-\chi_m f_{m-1}(\eta)]=\hbar_1 H_1(\eta)R_m^f(\eta) \tag{29}$$

$$L_\theta[\theta_m(\eta)-\chi_m\theta_{m-1}(\eta)]=\hbar_2 H_2(\eta)R_m^\theta(\eta) \tag{30}$$

Subject to the boundary conditions

$$f_m(0)=0 \qquad f_m'(0)=0 \qquad f_m'(\infty)=0$$

$$f_m''(\infty)=0 \qquad \theta_m(0)=0 \qquad \theta_m(\infty)=0 \tag{31}$$

$$R_m^f(\eta)=f_{m-1}'''+\sum_{i=0}^{m-1} f_i f_{m-1-i}''-2\sum_{i=0}^{m-1} f_i' f_{m-1-i}'-K_1\{3\sum_{i=0}^{m-1} f_{m-1-i}' f_i'''-\frac{1}{2}\sum_{i=0}^{m-1} f_{m-1-i} f_i^{iv}-\frac{3}{2}\sum_{i=0}^{m-1} f_{m-1-i}'' f_i''\} \\ -(k_2+M)f_{m-1}' \tag{32}$$

$$R_m^\theta(\eta)=\theta_{m-1}''+\Pr\sum_{i=0}^{m-1} f_i\theta_{m-1-i}'-2\Pr\sum_{i=0}^{m-1} f_i'\theta_{m-1-i}+k_2 Ec\Pr\sum_{i=0}^{m-1} f_i' f_{m-1-i}' Ec\Pr\sum_{i=0}^{m-1} f_i'' f_{m-1-i}'' \tag{33}$$

$$\chi_m=\begin{cases}0, m\le 1\\ 1, m>1\end{cases} \tag{34}$$

We choose the auxiliary function as

$$H_1(\eta)=1,\ H_2(\eta)=1 \tag{35}$$

If we let $f_m^*(\eta)$, $\theta_m^*(\eta)$ as the special functions of mth order deformation equations, then the

general solutions are given by

$$f_m(\eta)=f_m^*(\eta)+C_1+C_2e^{-\eta}+C_3e^{\eta} \tag{36}$$

$$\theta_m(\eta)=\theta_m^*(\eta)+C_4e^{-\eta}+C_5e^{\eta} \tag{37}$$

Where the integral constants C_i (i =1,2,3,4,5) are determined using the boundary conditions (31).

Now it is easy to solve the linear non-homogeneous equation (29) and (30) using MATHEMATICA software one after the other by considering m = 1, 2, 3…

4. CONVERGENCE OF HAM SOLUTION

By using Homotopy Analysis Method, the convergence region and rate of approximation of series solutions can be adjusted and controlled by using the non-zero auxiliary parameters $\hbar_1$ *and* $\hbar_2$. To find $\hbar_1$ *and* $\hbar_2$, $\hbar$ curves are plotted in figure 1 and figure 2 for 20th order of approximation. From figure 1 and figure 2 the range of admissible values

of $\hbar_1$ *and* $\hbar_2$ are about $-0.91 \le \hbar_1 \le -0.32$ and $-1.61 \le \hbar_2 \le -0.98$. The convergence of HAM solution for different orders of approximations is given in Table.1.

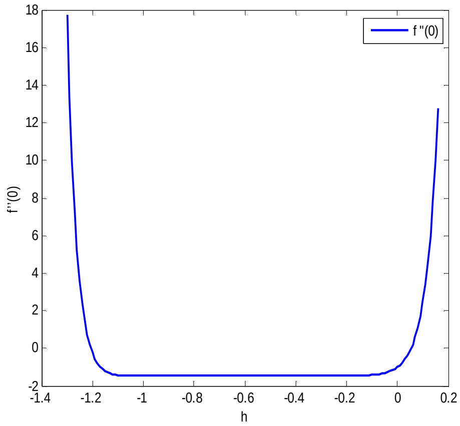

Figure 1: ħ-curve for f´´(0)

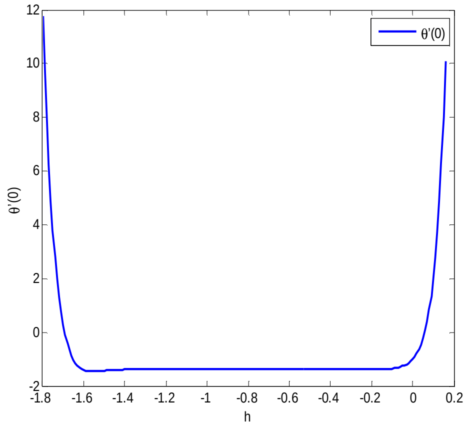

Figure 2: ħ-curve for θ´(0)

5. RESULTS AND DISCUSSIONS

In this paper, we analyze the heat transfer of MHD visco-elastic fluid flow in porous medium with viscous dissipation over an exponentially stretching sheet. The effects of visco-elastic parameter, porosity parameter, Magnetic parameter, Prandtl number and Eckert number on velocity and temperatures profiles are represented through graphs. Table.2 enlists the comparison of θ´(0) for different values of visco-elastic parameter k_1 and are compared with the available results in the literature and are seen in good agreement.

Table 1: Convergence of HAM Solution for Different Orders of Approximations when
$k_1 = 0.1,\ k_2 = 0.1,\ Ec = 0.1,\ \Pr = 1.0$

Order	$-f''(0)$	$-\theta'(0)$
5	1.424645	1.203512
10	1.427125	1.200031
15	1.427156	1.199619
20	1.427156	1.199521
25	1.427156	1.199492
30	1.427156	1.199483
35	1.427156	1.199481
40	1.427156	1.199481

Table 2: Comparison of –θ'(0) for Different Values of k_1 with k_2 = 0.5, Pr = 1.0, Ec = 0.25

K_1	Mahentesh M (13)	Present Study
0.0	1.12221	1.122209
0.1	1.09631	1.0963096
0.2	1.06551	1.065513

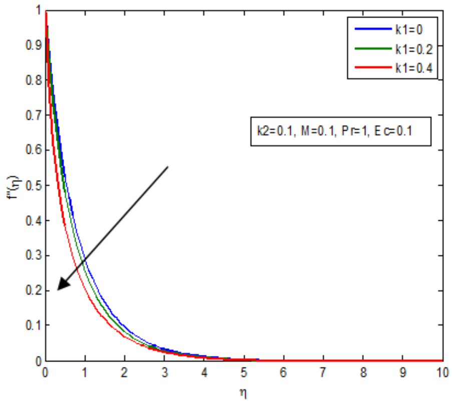

Figure3: Velocity f´ (η) for different Values of k_1

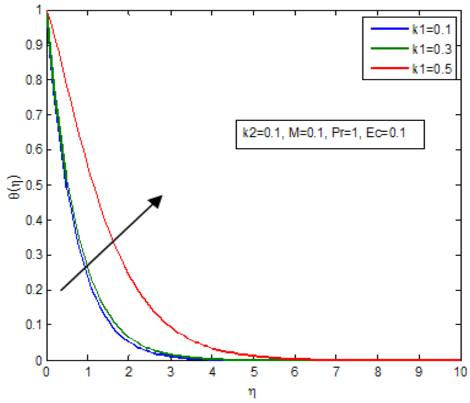

Figure4: Temperature θ (η) for different Values of k_1

The effect of visco-elastic parameter on velocity is shown in Figure 3. It can be shown that as viscoelastic parameter increases the velocity decreases i. e. the flow of the fluid retards with increasing of viscoelastic parameter. Figure 4 reveals the effect of visco-elastic parameter on temperature. It can be noticed that increase in viscoelastic parameter corresponds to an increase in temperature of the fluid. This is due to the fact that increase of visco-elastic normal stress causes the thickening of thermal boundary layer.

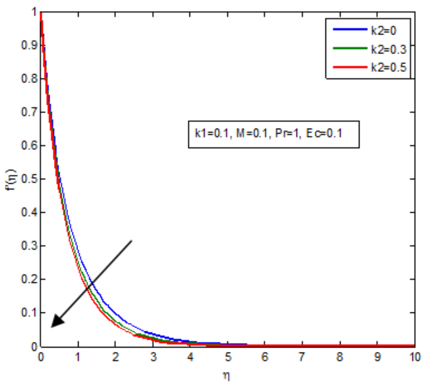

Figure 5: Velocity f´ (η) for different Values of k_2

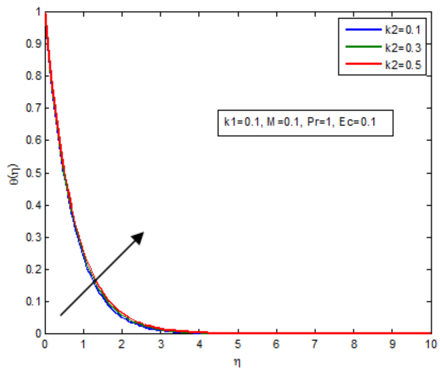

Figure 6: Temperature θ (η) for different Values of k_2

Figure 5 is drawn for different values of porosity parameter k_2 on velocity. From figure 5 the velocity decreases with increasing values of porosity parameter, which leads to enhanced deceleration of the flow. Figure 6 indicates the effect of porosity parameter on temperature. It can be shown that by increasing the values of porosity parameter, the temperature of the fluid and thermal boundary layer thickness increases.

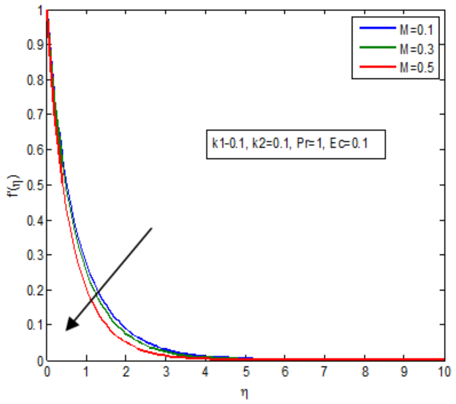

Figure 7: Velocity f´ (η) for different Values of M

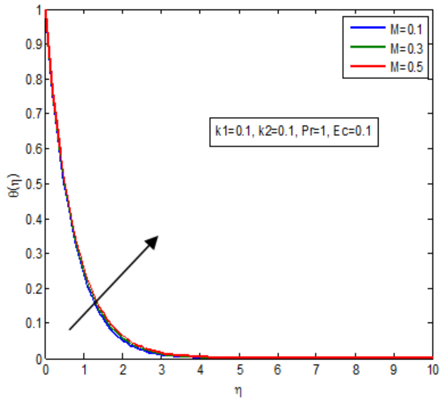

Figure 8: Temperature θ (η) for different Values of M

Figure 7 and 8 depicts the effect of magnetic parameter on velocity and temperature. It illustrates that by increasing the magnetic parameter the velocity decreases whereas temperature increases. With increase in values of M, rate of transport decreases due to the Lorentz force which opposes the motion of the fluid.

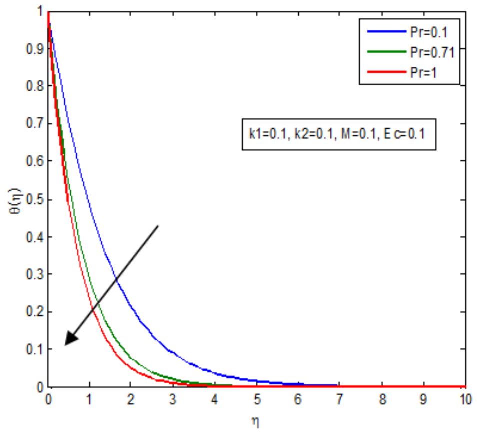

Figure 9: Temperature θ (η) for different Values of Pr

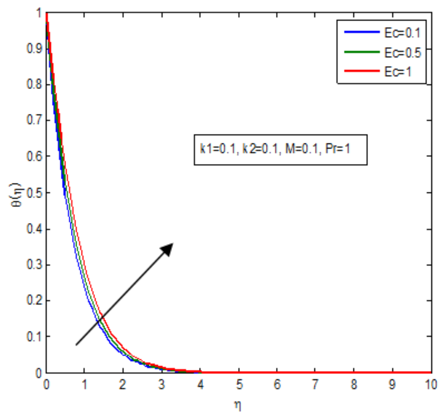

Figure 10: Temperature θ (η) for different Values of Ec

The effect of Prandtl number and Eckert number is shown in Figure 9 & 10. By increasing Prandtl number the temperature decreases whereas it increases by increasing Eckert number. With increasing values of Prandtl number the thermal boundary layer thickness decreases so that the diffusion of heat is reduced. By increasing the values of Eckert number the thermal boundary layer thickness increases. This raise in temperature occurs as heat energy is stored in the fluid due to frictional heating.

6. CONCLUSIONS

In this paper, we applied Homotopy Analysis Method to heat transfer of MHD visco-elastic fluid flow through porous medium over an exponentially stretching sheet with viscous dissipation. The effect of various physical parameters on velocity and temperature are discussed through graphs and the results are summarized as follows.

- Visco-elastic parameter, porosity parameter and Magnetic parameter have same effect on temperature. i. e. temperature increases with increasing values of parameters.

- Diffusion of heat is reduced with increasing values of Prandtl number.
- The thermal boundary layer thickness increases with increasing of Eckert number.
- For effective cooling of stretching sheet visco-elastic liquids having low viscous dissipation must be chosen.

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